Homework 5 - Solutions

(1)

$\boxed{\text{No}}$ - W is not a vector space

because the zero vector $\begin{bmatrix} 0 \\ 0 \end{bmatrix} \notin W$

since $0 \neq 2(0)+1$

(W is also not closed under addition or scalar multiplication.)

(2) Claim: The set of all Rational numbers is NOT a vector space.

Pf: The set of Rationals is not closed under scalar multiplication.

for example, 1 is Rational and $\sqrt{2}$ is a scalar in $\mathbb{R}$

$\sqrt{2} \cdot 1 = \sqrt{2} \notin$ Rationals

Hence, the Rational numbers do not make up a vector space. ∎

(3) $T: \mathbb{R}^2 \to \mathbb{R}^3$

$T(v) = 0$

Claim: T is a linear transformation.

Pf: let u, v be vectors in $\mathbb{R}^2$ and $c \in \mathbb{R}$

Then $T(u+v) = 0 = 0+0 = T(u)+T(v)$ ✓

$T(cu) = 0 = c(0) = cT(u)$ ✓

Therefore, by definition, T is a linear transformation. ∎

(4) $T\left(\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}\right) = \begin{bmatrix} -v_1 - 7v_3 \\ v_2 \\ v_1 \\ 0 \\ v_2 - v_3 \end{bmatrix}$ find A such that $T(v) = Av$

$$A = \begin{bmatrix} -1 & 0 & -7 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix}$$

(5) Row Reduce $\left[\begin{array}{cc|c} -4 & 1 & 2 \\ 7 & 2 & 3 \\ 8 & 3 & 1 \end{array}\right]$

$$\xrightarrow{-\frac{1}{4}R_1} \left[\begin{array}{cc|c} 1 & -\frac{1}{4} & -\frac{1}{2} \\ 7 & 2 & 3 \\ 8 & 3 & 1 \end{array}\right] \xrightarrow[-8R_1+R_3]{-7R_1+R_2} \left[\begin{array}{cc|c} 1 & -\frac{1}{4} & -\frac{1}{2} \\ 0 & \frac{15}{4} & \frac{13}{2} \\ 0 & 5 & 5 \end{array}\right]$$

$$\xrightarrow{\frac{4}{15}R_2} \left[\begin{array}{cc|c} 1 & -\frac{1}{4} & -\frac{1}{2} \\ 0 & 1 & \frac{26}{15} \\ 0 & 5 & 5 \end{array}\right] \xrightarrow[-5R_2+R_3]{\frac{1}{4}R_2+R_1} \left[\begin{array}{cc|c} 1 & 0 & -\frac{2}{30} \\ 0 & 1 & \frac{26}{15} \\ 0 & 0 & -\frac{11}{3} \end{array}\right]$$

$\Downarrow$

$0 = -\frac{11}{3}$ No Solution

thus $\boxed{\text{W is not in the Span}}$

6 a) $u \in \text{Nul } A$ if $A\vec{u} = \vec{0}$

$$\begin{bmatrix} 2 & 4 & -2 & 1 \\ -2 & -5 & 7 & 3 \\ 3 & 7 & -8 & 6 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -3 \\ 3 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\boxed{\text{No, } u \notin \text{Nul} A}$

b) if $\vec{u} \in \text{Col} A$ then $\vec{u} = A\vec{x}$ for some $\vec{x} \in \mathbb{R}^4$

$$\underset{4\times 1}{\begin{bmatrix} 3 \\ -2 \\ -1 \\ 0 \end{bmatrix}} \neq \underbrace{\underset{3\times 4}{\begin{bmatrix} 2 & 4 & -2 & 1 \\ -2 & -5 & 7 & 3 \\ 3 & 7 & 8 & 6 \end{bmatrix}} \underset{4\times 1}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}}_{\text{should give us a } 3\times 1}$$

$\boxed{\text{No, } \vec{u} \notin \text{Col} A}$ since Col A is made up of vectors in $\mathbb{R}^3$.